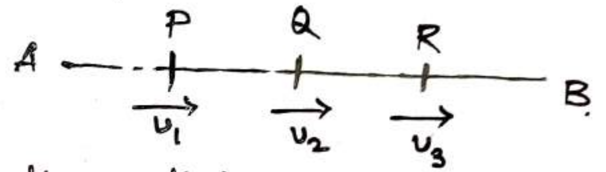
# Viscosity.

Both liquid and ~~solid~~ gas have fluid properties as their constructed molecules have freedom to move. This flowness of molecules causes bulk material to move and to diffuse. Though the flowness of liquid is differ from that of the gas. At a given temperature of certain mass the volume of a liquid is mostly constant. But for gases the volume is dependent on pressure and temperature. One similarity is that like gases, molecules in liquids have constant random motion and their kinetic energy is proportional to temperature. Viscosity plays an important role for the flowness of liquids and gases.

The flow of liquid may be ordered or random.

## Stream line Flow :-




When the velocity of the particles in a liquid flow-line exactly the same in ~~every~~ each points in its path of movement both in magnitude and direction. It is not necessary for all the particles, the velocity to be constant or same at its whole path.

Consider a tube AB through which liquid is flowing. Let the velocities of particle at P is $v_1$, at Q is $v_2$ and at R is $v_3$. If all other particles of the liquid pass through P with velocity $v_1$, Q with $v_2$ and R with $v_3$, ie their velocities remain unchanged at its path. it is said to be steady or streamline flow.

## Turbulent Flow :-

When the flow of liquid is disordered or not same at each points the flow is termed as turbulent.

The flow will be turbulent when a liquid moves with a velocity greater than its critical velocity.

The critical velocity is that limit of velocity of flow of liquid upto which the motion of the liquid is stream line. Sometimes it is mentioned by Reynolds number.

## Reynolds Number (Re)

The Reynolds number is the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities. A region where these forces change behaviour is known as a boundary layer. In mathematical form it can be written as

$$\boxed{Re = \frac{\rho u L}{\mu} = \frac{uL}{\nu}}$$

where $\rho$ = density of fluid [ $Kg/m^3$ ]

$u$ = flow speed [ $m/s$ ]

$L$ = characterstic linear dimension [ $m$ ]

$\mu$ = dynamic viscosity of the fluid [ $Pa.s$ or $N.s/m^2$ ]

$\nu$ = Kinematic viscosity of the fluid [ $m^2/s$ ].

$Re < 2300 \Rightarrow$ ~~turbulent~~ Laminar flow

$Re > 2900 \Rightarrow$ turbulent flow.

## Newtonian fluid

A ~~Newtonian~~ Newtonian fluid is a fluid which obey the Newton's Law of ~~motion~~ viscosity. An element of a flowing liquid or gas will suffer forces from the surrounding forces, including viscous stress forces that cause it to gradually deform over time. The mathematical form of Newton's law of viscosity is expressed as

$$\tau = \mu \frac{dv}{dr}$$

Where $\tau$ = shear stress (drag)

$\mu$ = Shear viscosity of fluid

$\frac{dv}{dr}$ = derivative of velocity

**Another form:-**

According to Newton

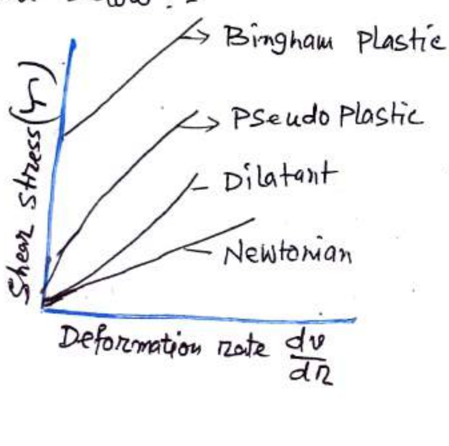
$$f \propto A \qquad A = \text{area.}$$

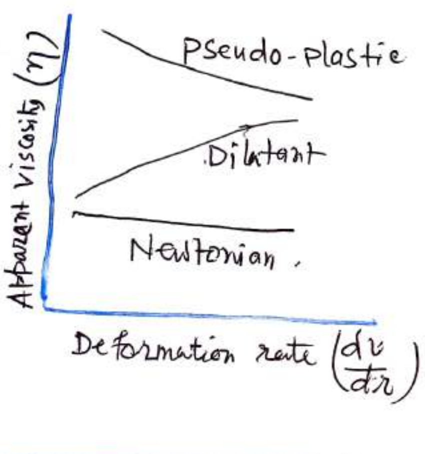
$$\text{and } f \propto \frac{dv}{dr} = \text{velocity gradient}$$

$$\text{or } \boxed{f = -\eta A \cdot \frac{dv}{dr}} \qquad \eta = \text{viscosity.}$$

For most non-Polymeric liquids $\eta$ is independent of $\frac{dv}{dr}$, such liquids are called Newtonian liquids.

Polymeric and colloidal solutions are non-Newtonian. Graphical representation of some fluids are given below:-



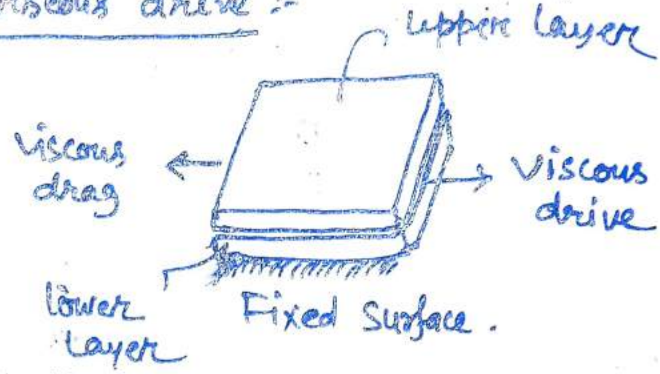



Example

Newtonian fluid — water, air, milk, alcohol, glycerol etc.

Non-Newtonian fluid — Blood, Toothpaste, Custard, Ketchup etc.

Viscous-drag and viscous drive :-




The viscous drag is a kind of friction or fluid resistance, acting relatively to the opposite of motion of fluid, This can exist between two layers or a fluid and a solid surface. The drag force is proportional to the velocity for the laminar flow and squared velocity for a turbulent flow.

The drag depends on the properties of the fluid and on the size, shape and speed of the object, The drag equation is

$$\boxed{F_D = \frac{1}{2} \rho v^2 C_D \cdot A}$$

$F_D$ = drag force
$\rho$ = density of fluid
$v$ = speed
$A$ = cross-sectional area
$C_D$ = drag-coefficient, it is diamensionless number.

The property of a fluid by virtue of which a retarding force is called into play whenever there occurs a relative motion between its different layers is called viscosity. This retarding force is called a 'viscous drag'. The flow through the tube occurs due to pressure or concentration gradient along the tube. The force which must be applied to maintain this pressure gradient for steady-flow of fluid is called 'viscous drive'

Unit :-

$$F = -\eta A \cdot \frac{dv}{dr} \qquad \eta : \text{Co-eff of viscosity}$$

when $A = 1\,cm^2$, $dv = 1\,cm\,sec^{-1}$, $dr = 1\,cm$

$$F = \eta \text{ dynes.}$$

Thus $\eta$ may be defined as the force per unit area required to maintain a velocity difference of 1 unit per sec between two parallel layers of fluid held at an ~~distance of~~ unit distance from each other.

In C.G.S

$$\eta = \frac{\text{dynes}}{cm^2} \times \frac{cm}{cm\,sec^{-1}} = \boxed{\text{dynes s } cm^{-1}}$$

|| Poise.

In SI.

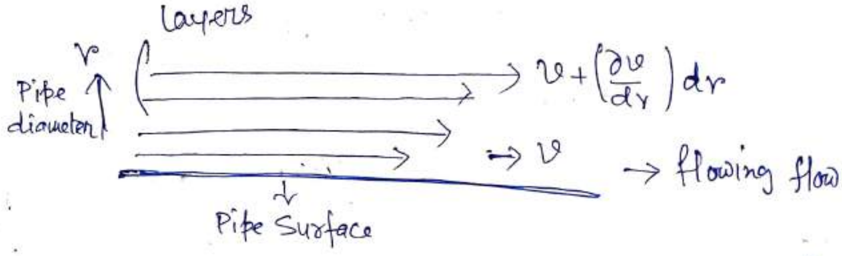
$$\eta = N m^{-2} s \text{ or Pa.s or Poiseullie.}$$

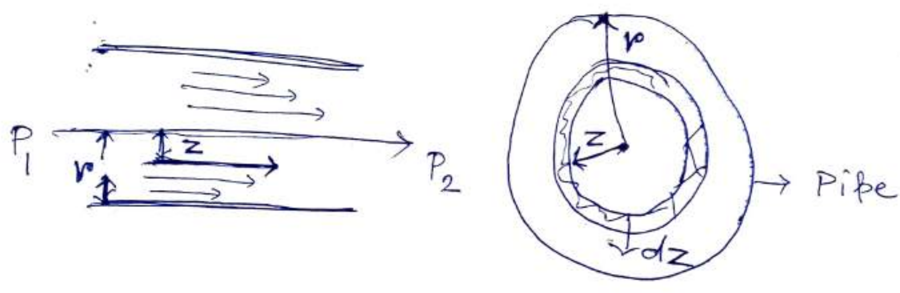
$$1 \text{ Poise} = 1\,gm\,cm^{-1}\,sec$$
$$= 10^{-3}\,Kg \times 10^{2}\,m^{-1}\,sec$$
$$= \cdot 1\,Kg\,m^{-1}\,sec$$
$$= \cdot 1\,Pa \cdot s$$
$$= \cdot 1 \text{ Poiseullie (Pl)}$$

# Poiseuille's equation:

For an incompressible, Newtonian, laminar flow of a liquid/gas through a long uniform cross-sectioned pipe the difference in pressure between the two ends measures by Poiseuille equation. Or on the other hand by knowing the pressure difference viscosity of the flowing flow is ~~~~ determined by ~~Poiseuille~~ Poiseuille equation.. For larger diameter of the pipe there is a chance for the flow to be turbulent.

Let us consider a narrow tube of uniform cross-section through which the fluid is flowing. Consider the fluid as an assemble of very thin layer one of the layer is in contact with pipe over which another layer over another and so on.



Now if we consider it as liquid flow, the liquid which is in contact with the pipe is stationary and the velocity of the layers increases with increasing distance from the surface. That means the velocity of the layer which is in the middle is maximum. If the velocity gradient is $\frac{dv}{dr}$ and area of contact is A then the internal frictional force(f) which is dragging the liquid to flow, as per Newton

$$f \propto A \;;\; f \propto -\frac{dv}{dr} \quad \text{or} \quad \boxed{f = -\eta . A . \frac{dv}{dr}}$$

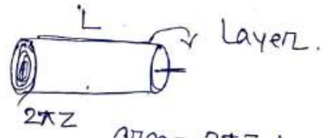
Now, let the radius of the pipe = r

Length of the pipe = L

Let v be the velocity at a distance Z from the centre, then

$$f = -\eta . A . \frac{dv}{dZ}$$

$$= -\eta . (2\pi . L . Z) . \frac{dv}{dZ}$$




area = $2\pi Z . L$

For the steady rate of flow, this frictional force is exactly the same with the driving force $\pi Z^2 (P_1 - P_2)$ where $(P_1 - P_2)$ is the pressure difference of the two ends of the pipe.

Therefore, $\pi Z^2 (P_1 - P_2) = -\eta (2\pi Z . L) \frac{dv}{dZ}$

$$\Rightarrow \cancel{\pi} Z . (P_1 - P_2) = -\eta . 2\cancel{\pi} L . \frac{dv}{dZ}$$

$$\text{or, } dv = -\frac{P_1 - P_2}{2\eta L} . Z \, dZ$$

on integration

$$\int dv = -\int \frac{(P_1 - P_2)}{2\eta L} z\, dz$$

or, $v = -\frac{P_1 - P_2}{4\eta L} \cdot z^2 + c$

When $v = 0$, $z = r$ [ as velocity is zero at the surface ].

or, $c = \frac{P_1 - P_2}{4\eta L} \cdot r^2$

or $v = -\frac{P_1 - P_2}{4\eta L} z^2 + \frac{P_1 - P_2}{4\eta L} \cdot r^2$

$= \frac{P_1 - P_2}{4\eta L} (r^2 - z^2)$.

Now,

Total volume of liquid flowing through the tube per unit time is given by

$$\frac{dV}{dt} = \int_0^r 2\pi z \cdot v \cdot dz$$

$$= \int_0^r 2\pi z \cdot \frac{P_1 - P_2}{4\eta L} \cdot (r^2 - z^2) \cdot dz$$

$$= \frac{\pi (P_1 - P_2) r^4}{8\eta L}$$

or, $\boxed{\eta = \frac{\pi (P_1 - P_2) r^4}{8L \cdot \left(\frac{dV}{dt}\right)}}$

This is poiseuille's equation.

For gaseous flow this equation comes with slight modification. Unlike the liquid the flowing of gas molecules through the tube the volume of it per second at any cross-section of the tube is not constant as it is not uncompressible like liquids, but the mass is constant. So if $V$ denotes the volume of the gas flowing the tube per second at a distance $x$ from the entry point of the tube, $\rho$ is the density at that section, then pressure $P$ over that section, then

$$\rho V = \text{Constant}$$

$$\text{or } PV = \text{Constant} \quad [\text{Since } \rho \propto P]$$

Considering a small cross section $dx$ at a distance $x$ from the entrance point with a pressure difference $dP$ accross it, then by Poiseuille formula

$$V = -\frac{\pi r^4}{8\eta} \cdot \frac{dP}{dx}$$

$$\text{or, } PV = -\frac{\pi r^4}{8\eta} \cdot P \cdot \frac{dP}{dx} \quad [\text{Multiplying by } P]$$

Now, if $P_1$ be the pressure at which the gas enters the tube and $V_1$ is the volume of the gas, then

$$P_1 V_1 = PV = -\frac{\pi r^4}{8\eta} P \cdot \frac{dP}{dx}$$

On integration

$$\int P_1 V_1 \, dx = -\int_{P_1}^{P_2} \frac{\pi r^4}{8\eta} P\, dP \quad \left[\begin{array}{l} L = \text{Length} \\ P_2 = \text{ends Pressure} \end{array}\right]$$

$$\text{or, } \boxed{P_1 V_1 = \frac{\pi r^4}{16 \eta L} \cdot (P_1^2 - P_2^2)}$$

$$\text{or, } \boxed{\eta = \frac{\pi r^4}{16 L P_1 V_1} (P_1^2 - P_2^2)} \rightarrow \text{This eq}^n \text{ is for gas-flow.}$$

* **N.B:-**

$$F = -\eta \cdot A \cdot \frac{dv}{dx}$$

If we measure the co-efficient of viscosity from this equation i.e. the absolute viscosity coefficient, some-time it is mentioned as dynamic viscosity co-efficient. If we devide it with the density it is called Kinematic viscosity co-efficient. C.G.S unit of Kinematic viscosity co-efficient is Stokes (St).

$$1\ St = 1\ cm^2 s^{-1}$$

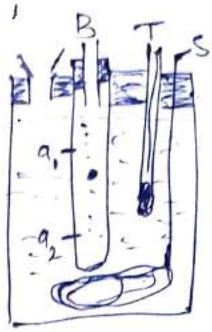
$$1\ St = \frac{1\ Poise}{g \cdot cm^{-3}} = \frac{dyne \cdot cm^{-2} \cdot s}{g \cdot cm^{-3}} = \frac{g\ cm \cdot s^{-2} \cdot cm^{-2} \cdot s}{g \cdot cm^{-3}} = \underline{cm^2 s^{-1}}$$

Kinematic viscosity co-efficient is sometimes referred to as diffusivity of momentum.

Kinematic viscosity of $H_2O$ at $20^\circ C \sim 1\ cSt$ (centi Stokes)

* **Determination of viscosity coefficient by falling sphere method:-**

When a metallic spherical ball of radius 'a' falls freely through a liquid, its velocity becomes constant, which is called terminal velocity ($v_t$).




This type of apparatus is used to determined the viscosity of liquid by falling sphere method. In this experiment the tube is filled with the liquid whose viscosity coefficient is to be determined. There is a stirrer 'S' and a thermometer 'T'. A small ball of radius 1-2 mm ~~are~~ is dropped through the narrow tube 'B'. The time 't' taken by the ball to travel from $a_1$ to $a_2$. Now the terminal velocity, ($v_t$) -

$$v_t = \frac{\overline{a_1 a_2}}{t}$$

By knowing the radius of the ball, densitites of ball and liquid, the viscosity co-efficient can be measured by the following way

When the terminal velocity is attained there will be no unbalanced force. Then the downward force of the ball will be due to its weight only. So,

$$F\downarrow = mg = \frac{4}{3}\pi a^3 \cdot \rho_s \cdot g \quad [a = \text{radius}]$$

Upward force is due to the hydro-static force and the viscous force. Thus

$$F\uparrow = \text{hydrostatic force} + \text{viscous force}$$
$$= \underbrace{\frac{4}{3}\pi a^3 \rho_l \cdot g}_{\text{(weight of liquid)}} + \underbrace{6\pi a \eta v_t}_{\text{(Stokes law)}}$$

At Steady-state $F\uparrow = F\downarrow$

$$\frac{4}{3}\pi a^3 \rho_L \cdot g + 6\pi a \eta v_t = \frac{4}{3}\pi a^3 \rho_s \cdot g$$

$$\text{or, } 6\pi a \eta v_t = \frac{4}{3}\pi a^3 g (\rho_s - \rho_L)$$

$$\text{or, } \boxed{\eta = \frac{2}{9}\frac{a^2}{v_t}(\rho_s - \rho_L)\cdot g}.$$

The wall of the tube however influence the velocity and a correction term is employed. Thus

$$\eta = \frac{2}{9}\frac{a^2(\rho_s - \rho_L)g}{v_t\left[1 + 2.1\left(a/R\right)\right]}$$

Where $R$ = Radius of the Container tube.

* A raindrop falling with uniform acceleration (due to gravity) has a downword force 'mg'. This downward force is balanced by the buoyancy of air and also by opposing viscous drag. Finally these two opposing forces balance each other and the raindrop falls freely with a constant velocity, which is the terminal velocity.

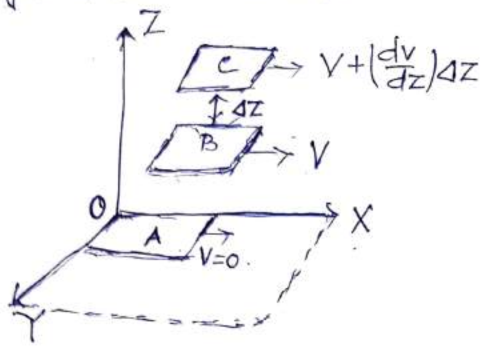
## Viscosity and Mean free Path ($l$) :-

The mean free path is the avarage distance covered by a molecule between two successive collisions. If the avarage velocity of the molecule is $c_a$ and $Z_1$ no. of collisions it suffers then the mean free path ($l$) of the molecule can be expressed as

$$l = \frac{c_a}{Z_1} = \frac{c_a}{\sqrt{2}\pi\sigma^2 c_a n} \qquad \left[\begin{array}{l} n = \text{no. of molecules per cc} \\ \sigma = \text{effective diameter} \end{array}\right]$$

$$= \frac{1}{\sqrt{2}\pi\sigma^2 n}$$

Let us consider a gas flow in layers.




Let the gas is flowing in layers parallel to XY-Plane. Let A, B and C are such layers. Layer A is in contact with XY-Surface. So the velocity would be zero at that plane. With increasing distance from XY-Plane in the Z-direction the velocity of the layers will also increase.

Now let us consider two layers B and C. Obviously Layer C will have more velocity than layer B.

let the velocity of layer B is $V$

" " " " $C = V + \left(\frac{dv}{dz}\right)\Delta z$,

where $\frac{dv}{dz}$ is the velocity gradient,

$\Delta z$ is the avarage distance between B and C.

Now if '$l$' be the mean free path of the gas molecule, then $\Delta z$ will be replaced by '$l$'.

Apart from the laminar flow of the gas due to an external pressure, the molecules will also have the thermal motion. Again as the molecular bond is not so strong between molecules they will inter-change between layers.

Now molecules in layer B will go to the higher level C will less momentum (as they have less velocity but mass is same). While those are coming from layer C to layer B will carry more momentum. The result is that the avarage velocities of the molecules of the upper layer will decrease and that of lower level will increase. This transport of momentum of the gas molecules is responsible for the viscosity of the gas.

Consider the layer B. The molecules which comes from lower of B will have velocities $= v - l\left(\frac{\partial v}{\partial z}\right)$ which comes above of B will have velocities $= v + l\left(\frac{\partial v}{\partial z}\right)$.

As there are X, Y and Z directions, gas molecules can move any of the directions, so taking the Probability only $1/3$ of molecules will move up and down in the Z-axis.

Out of these $1/3$ moleculers, again considering the Probability $1/2$ of it will move upwards and another $1/2$ will go downwards.

So we have $1/6$ molecules.

Now if the cross-section of the gas-layer is 's', the number of molecules will go downwards $= \frac{1}{6} c_a . n . s$ [ $c_a$ = avarage velocity
$n$ = no. of molecules per cc ].

So, the momentum transported to the layer 'B' per second from above $= \frac{1}{6} m . c_a . n . s \left[ v + \left(\frac{\partial v}{\partial z}\right) . l \right]$.

Similarly momentum transported to the layer 'B' per second from below $= \frac{1}{6} m . c_a . n . s . \left[ v - \left(\frac{\partial v}{\partial z}\right) l \right]$.

Thus net momentum transported per second to the layer 'B'

$$= \frac{1}{6} m c_a n s \left[ v + \left(\frac{\partial v}{\partial z}\right) l \right] - \frac{1}{6} m c_a n s \left[ v - \left(\frac{\partial v}{\partial z}\right) l \right].$$

$$= \frac{1}{6} . m . c_a n s \left[ 2 . \left(\frac{\partial v}{\partial z}\right) l \right]$$

$$= \frac{1}{3} m c_a n s l \left(\frac{\partial v}{\partial z}\right).$$

This momentum-change per second is the internal frictional force $= \eta . s . \left(\frac{\partial v}{\partial z}\right) \Rightarrow$ [From Newton's law of viscosity]

Thus $\eta . s . \left(\frac{\partial v}{\partial z}\right) = \frac{1}{3} m n c_a s l \left(\frac{\partial v}{\partial z}\right)$

or, $\eta = \frac{1}{3} m n c_a . l$. [ $m$ = mass of the molecule
$\rho$ = density ).

$= \boxed{\frac{1}{3} \rho c_a l}$

i.e $\boxed{\eta = \frac{1}{3} \rho c_a l}$.

Now, $\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 n}$

So, $\eta = \frac{1}{3} \rho . c_a . \frac{1}{\sqrt{2}\pi\sigma^2 n}$.

$= \frac{1}{3} m.n. c_a . \frac{1}{\sqrt{2}\pi\sigma^2 n} = \boxed{\frac{m.c_a}{3\sqrt{2}\pi\sigma^2}}$.

~~Ag~~. From this relation we can conclude the $\eta$ is independent of Pressure.

Again $c_a = \left(\frac{8KT}{\pi m}\right)^{1/2}$

So, $\eta = \frac{m}{3\sqrt{2}\pi\sigma^2} \cdot \left(\frac{8KT}{\pi m}\right)^{1/2}$

$= \frac{m^{1/2} . 2\sqrt{2} (KT)^{1/2}}{3\sqrt{2}\pi^{3/2} . \sigma^2}$

$= \frac{2}{3} \frac{(mKT)^{1/2}}{\sigma^2 \pi^{3/2}} = \frac{2\sqrt{RTM}}{3\pi^{3/2} N_0 . \sigma^2}$

ie $\boxed{\eta \propto \sqrt{T}}$

Thus viscosity of the gas is proportional to the square root of the absolute temperature.

~~However~~. However the experiments show some higher value. This is due to the existance of intermolecular attraction which is not taken into account. An empirical relation about this is

$\eta = \frac{K . \sqrt{T}}{1 + c/T}$ where, K, c are constants

This is Sutherland equation. 'C' is called Sutherland constant.